

Partial restoration of factorization and universality in presence of factorization breaking interactions in hadronic hard scattering processes.

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Recent works have discussed the violation of factorization and universality in hadronic hard scattering processes aimed at measurements of T-odd distributions. We use simple arguments to show that it is possible to restore an approximate factorization involving T-odd contributions, if the factorization breaking interactions present a frequency spectrum dominated by a narrow and regular peak whose maximum value corresponds to a respected factorization.

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I. INTRODUCTION

Recent works[1, 2] have seriously re-discussed the ideas of factorization and universality in hard processes aimed at the detection of T-odd distributions. In the last fifteen years the study of these distributions has often made it necessary to reconsider the basic properties of the QCD-improved parton model, in particular factorization[3, 4].

In quark/parton physics the first T-odd distribution function was probably the Sivers function[5] in 1990, followed by the Boer-Mulders-Tangerman function[6, 7, 8]. The former was used to explain single spin asymmetries[9], the latter to explain unpolarized Drell-Yan azimuthal asymmetries[8]. Recently, it has been demonstrated[10] that the T-odd mechanism introduced in[11] and [12] produces a Sivers asymmetry when its effect is extrapolated into the small transverse momentum region. Also, in high energy nuclear physics a T-odd structure function, the so-called “fifth structure function”, was introduced[13] and modeled[14, 15] to describe normal asymmetries in $A(\vec{e}, \vec{e}'p)$ quasi-elastic scattering.

For justifying the same existence of leading twist T-odd functions, the central object is the gauge restoring operator entering the definition of parton distribution[3]. In an approach where factorization is assumed from the very beginning, this gauge field is assumed to be negligible at leading twist, but in its absence T-odd distributions are forbidden[16] by general invariance principles. A chain of arguments and examples[17, 18, 19, 20, 21, 22] has shown that proper taking into account the gauge factor permits the existence of leading twist T-odd distributions in a QCD framework, associated with formal factorization breaking by interactions in the initial or final state. In the same works it has been shown that in some cases it is possible to recover factorization from a practical point of view. Systematic efforts have been undertaken[10, 23] for rewriting factorization rules for processes where partonic distributions depend on moderate values of the transverse momentum and the gauge factor is assumed to play a role.

Several models and studies have been published on T-odd distributions[24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34], together with phenomenological parameterizations of the Sivers function[35, 36, 37, 38] and of the Boer-Mulders-Tangerman function[8, 39]. So, we may say that T-odd distributions are an accepted reality, although we have no data yet about their behavior at asymptotic energies.

This creates the problem of the coexistence of factorization breaking and factorization, i.e. at which extent it is possible to reconcile the interpretation of observable effects in terms of factorization breaking interactions with the safety of the extremely simple and useful factorization/universality scheme for classifying phenomenology. Also, it raises questions[17, 19] on the probabilistic interpretation of the measured distributions. Most of the above quoted models assume, more or less explicitly, that the factorization breaking effects are there but small enough not to overthrow the underlying parton model picture. The analysis of [1] and the observations by [2] suggest that this point is still to be clarified, especially in hadron-hadron single spin asymmetries.

Without considering a model in detail, we show here that it is possible to restore an approximate factorization scheme involving the presence of nonzero T-odd distribution functions, if factorization breaking interactions satisfy certain restrictive conditions.

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The central object of any distribution function calculation is the imaginary part of an amplitude $G(2,1)$ that describes the formation of a quark/hole pair at a spacetime point 1, and the propagation of the hole up to the point 2 where the hole is filled by the quark again. A value of the longitudinal fraction x is associated with this quark hole. By definition $x(1) = x(2)$. In absence of factorization breaking interactions, this is true also in local sense: the quark hole conserves its initial longitudinal fraction along all the path from 1 to 2.

Factorization breaking interactions spoil this property. In any precise model treatment of rescattering, one includes explicitly degrees of freedom that exchange $O(Q)$ amounts of momentum/energy with the quark hole (see e.g. the calculations in ref.[10]), and consequently the wavefunction describing the quark hole contains plane wave components associated to values $x' \neq x$. A way to quantify the effectiveness of rescattering is to look at the size of the difference $x - x'$.

Our starting point is the idea that a rescattering mechanism produces a finite but *small* violation of x -conservation. To formalize this, in section IV we introduce the x -frequency spectrum $f_\epsilon(x - x')$ of the rescattering operator and require a set of properties for it. Qualitatively, we may write since now that we require $f_\epsilon(x - x')$ to consist of a regular and narrow peak at $x = x'$, with finite width $\epsilon \ll 1$. In addition, we require $f_\epsilon(x - x')$ to respect causality (see section IV for details).

The ϵ parameter is supposed to depend slowly on x and on $|k_T|$ (\vec{k}_T is the quark transverse momentum), and it expresses the magnitude of x -nonconservation in rescattering: $|x - x'| \sim \epsilon$. In the limit where $\epsilon \rightarrow 0$, rescattering does not affect the quark longitudinal fraction, and factorization is fully respected.

That the frequency spectrum is large for $x' \approx x$, and small when x and x' are very different, is suggested by the fact that at a certain extent factorization works empirically. The assumption that the peak of $f_\epsilon(x - x')$ at $x' = x$ is regular is possibly not justified for individual perturbative diagrams, but becomes more reasonable when we consider resummed sets of diagrams, or approximations where some degrees of freedom have been integrated over, or diagrams that are regularized via cutoffs, form factors, and so on. Clearly, the chosen values for model parameters may critically influence the size of the relevant region of the frequency spectrum. As above written, here we suppose that this size ϵ is small.

If the rescattering operator frequency spectrum satisfies the requested properties, on the ground of simple arguments we show that it is possible to write a transverse momentum dependent quark distribution in the form $q_0 + \epsilon q_1 + O(\epsilon^2)$, where q_0 may be calculated in complete absence of factorization breaking terms, and q_1 is a T-odd correction.

This restores approximate universality for the T-even term: for values of ϵ up to 0.3 it may be rather difficult to detect deviations from the q_0 behavior. This also agrees with the known complete absence of screening effects in hadron-nucleus Drell-Yan (see [40] for a review of the related experiments).

On the contrary, the T-odd correction term is potentially process-dependent unless one is able to demonstrate that ϵ has universal features. An effort in this direction has been undertaken elsewhere[34] by the author of the present work, but the arguments presented here are independent from the analysis in [34] and so we do not put constraints on ϵ in the following.

As a last remark, the enlarged-delta approximation[15] used in the following was developed to quickly approximate interactions between a pointlike particle and a composite system coexisting at the same time. Although it is probably possible to re-examine it, and to extend its validity to the case where the interacting particles exist at different times (the case of lepton-hadron SIDIS, and interactions between initial and final hadronic systems in hadron-hadron SIDIS), the present analysis is limited to the case of Drell-Yan and hadron-hadron initial state interactions.

II. SOME GENERAL DEFINITIONS AND NOTATIONS

We work, as far as possible, in the traditional picture of a distribution function, where a quark/hole pair is formed in a proton state $|P\rangle$ at a spacetime point “1” $\equiv (0, 0, 0, 0)$, and the hole propagates up to a point “2” $\equiv (0, z_-, \vec{b})$, where the quark is restored with the same momentum k_μ it had in the origin. The imaginary part of this amplitude gives, by suitable Dirac projections, all the distribution functions we need.

The quark/hole longitudinal fraction x is defined by $P_+ = xk_+$, where P_+ and k_+ are the large light-cone momentum components of the parent proton and of the quark. The infinite momentum limit $P_+ \rightarrow \infty$ selects leading twist contributions. Not to work with singularities, we introduce the scaled longitudinal variable ξ :

$$\xi \equiv P_+ z_-, \quad (1)$$

and a distribution function may be written as

$$q(x, \vec{k}_T) \equiv P_+ \int e^{-ixP_+ z_-} e^{ik_T b} g(z_-, \vec{k}_T) dz_- d\vec{k}_T \equiv \int e^{-ix\xi} e^{ik_T b} G(\xi, \vec{b}) d\xi d\vec{k}_T. \quad (2)$$

The range of useful values of ξ remains finite $\sim 1/x$ when $P_+ \rightarrow \infty$. So in this limit the function $G(\xi, \vec{b})$ does not need to be singular in the origin to produce a nonzero $q(x)$.

The coordinate z_+ plays no role in the following and is not explicitly reported anymore.

III. ENLARGED δ FUNCTION

We adapt here a technique from refs. [15] and [41] to roughly but quickly approximate the resummed effect of rescattering. Rescattering obliges us to include nonlocality with respect to x , involving integrals of the kind $\int dx' f(x, x')h(x')$ where the kernel $f(x, x')$ becomes $\delta(x - x')$ in the limit of respected factorization. In this work we consider kernels that allow for a *small* x -nonlocality: $|x - x'| \sim \epsilon \ll 1$.

Given a small nonzero parameter $\epsilon \ll 1$, we define an “ ϵ -enlarged δ -function” $\delta_\epsilon(x - x')$ such as to satisfy:

- (i) $x' \approx x$ within uncertainty $|\epsilon|$,
- (ii) $\int_{-\infty}^{\infty} dx' \delta_\epsilon(x - x') = 1$.

To define δ_ϵ in the limited x -range $[0 - 1]$, (ii) is better modified into:

(ii b) $1 - \alpha\epsilon^2 < \int_0^1 dx' \delta_\epsilon(x - x') < 1$, with $\alpha \sim 1$. This property is valid only for $x > \epsilon$ and $x < 1 - \epsilon$, so values of x near 0 and 1 will not be considered in this work.

Such functions are normally used to define the ordinary $\delta(x - x')$ function in the $\epsilon \rightarrow 0$ limit. Several functional choices for $\delta_\epsilon(x - x')$ are possible, the most useful for us is (for $\epsilon > 0$, else $\epsilon \rightarrow |\epsilon|$):

$$\begin{aligned} \delta_\epsilon(x - x') &\equiv \frac{1}{\pi} \frac{\epsilon}{\epsilon^2 + (x - x')^2} \\ &\equiv \delta_{\epsilon+}(x - x') + \delta_{\epsilon-}(x - x') \equiv \frac{i}{2\pi} \frac{1}{x - x' + i\epsilon} - \frac{i}{2\pi} \frac{1}{x - x' - i\epsilon}. \end{aligned} \quad (3)$$

Causality implies the substitution

$$\delta_\epsilon(x - x') \rightarrow \delta_{\epsilon\pm}(x - x'). \quad (4)$$

that corresponds, in its Fourier transform, to

$$\exp(-\epsilon|\xi|) \rightarrow \theta(\pm\xi)\exp(\mp\epsilon\xi) \quad (5)$$

Since all the relations interesting us contain a function $\theta(\xi)$ related with causality, δ_ϵ actually means $\delta_{\epsilon+}$, i.e. one pole only is encircled in the complex plane integrations.

We are especially interested in the calculation of integrals of the form $\int_0^1 dx' f(x')\delta_{\epsilon+}(x - x')$. For $\epsilon \ll 1$, and x far from its kinematic limits 0 and 1 ($x > \epsilon$ and $1 - x > \epsilon$) such integrals may be assumed to be dominated by the $\delta_{\epsilon+}$ pole $x' = x + i\epsilon$.

More precisely, we map the $[0, 1]$ x -range onto the $[-\infty, +\infty]$ z -range by the transformation

$$x \equiv \frac{e^z}{1 + e^z} \leftrightarrow z \equiv \ln(x/1 - x) \quad (6)$$

Since $dx = e^z/(1 + e^z)^2 dz$, and for x near the complex point a we have $1/(x - a) \approx (1 + e^z)^2/e^z \cdot 1/(z - A)$ (where $A \equiv z(a)$) we have

$$\frac{dx}{x - a} \approx \frac{dz}{z - A} \quad \text{for } x \approx a. \quad (7)$$

If the integral $\int_0^1 f(x')\delta_{\epsilon+}(x - x')$ is dominated by the $x' = x' - i\epsilon$ singularity we have:

$$\int_0^1 f(x')\delta_{\epsilon+}(x - x')dx' = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{f[x'(z')]}{[z' - z(x + i\epsilon)]} dz' \quad (8)$$

$$= f(x + i\epsilon) = f(x) + i\epsilon \frac{df}{dx} + O(\epsilon^2). \quad (9)$$

IV. INSERTION OF FACTORIZATION BREAKING INTERACTIONS

Let us define

$$q_0(x) \equiv \int d\xi e^{-ix\xi} \langle P | \psi(\xi) \psi^+(0) | P \rangle, \quad (10)$$

$$q_\epsilon(x) \equiv \int d\xi e^{-ix\xi} \langle P | \psi(\xi) F_\epsilon(\xi) \psi^+(0) | P \rangle. \quad (11)$$

Of the previous two equations, the former defines a parton-model leading twist quark distribution, the latter includes the effect of a rescattering operator $F_\epsilon(\xi)$, where a parameter ϵ quantifying the strength of F is explicitly reported. For $\epsilon = 0$, F_ϵ reduces to unity, and q_ϵ to the parton model definition.

We assume that F_ϵ is scalar, only to simplify the following relations (else, a double sum is present, see the end of this section). We introduce the frequency spectrum $f_\epsilon(x - x')$:

$$F_\epsilon(\xi) \equiv \int dy e^{-iy\xi} f_\epsilon(y), \quad -1 < y < 1. \quad (12)$$

For $f_\epsilon(y)$ we assume the following properties:

- 1) Although $f_\epsilon(y)$ is not a delta function, it consists of a peak at $y = 0$.
- 2) This peak is regular on the real axis.
- 3) The width ϵ of the peak is small: $\epsilon \ll 1$.
- 4) $f_\epsilon(y)$ is negligible for $|y| \gg \epsilon$.
- 5) For $\epsilon \rightarrow 0$, $f_\epsilon(y) \rightarrow \delta(y)$.
- 6) The shape of this frequency spectrum is causality-modified, so that $F(\xi)$ contains the factor $\theta(\xi)$. In other words, $f_\epsilon(y)$ is not an ordinary function but a distribution, associated with given integration rules.

The previous hypotheses allow one to approximate $f_\epsilon(y)$ the following way:

$$f_\epsilon(y) \approx \delta_{\epsilon+}(y). \quad (13)$$

Now we may write

$$q_\epsilon(x) = \int d\xi e^{-ix\xi} \langle P | \psi(\xi) \int dy e^{-iy\xi} \delta_{\epsilon+}(y) \psi^+(0) | P \rangle = \quad (14)$$

$$= \int dy \delta_{\epsilon+}(y) \int d\xi e^{-i(x+y)\xi} \langle P | \psi(\xi) \psi^+(0) | P \rangle = \quad (15)$$

$$= \int dy \delta_{\epsilon+}(y) q_0(x+y) = \int dy \delta_{\epsilon+}(x-y) q_0(y). \quad (16)$$

Applying the approximation eq.9 we get

$$q_\epsilon(x) \approx q_0(x + i\epsilon) \approx q_0(x) + i\epsilon \frac{dq_0}{dx} + O(\epsilon^2). \quad (17)$$

If one wants to be more precise and consider the non-scalar operator nature of F_ϵ , one may introduce explicitly the matrix sum $\langle P | \sum_{i,j} \dots | P \rangle$, and apply the previous considerations to each of the terms of the sum.

V. DETECTION OF THE FINITE IMAGINARY PART OF THE AMPLITUDE POLE AS A T-ODD DISTRIBUTION

For making the above imaginary shift observable in the form of a T-odd \vec{k}_T -dependent distribution function, factorization breaking interactions must also produce nonzero interference between orbital states differing by one unit.

So, we apply the considerations of the previous subsection to the impact parameter dependent distribution $q(x, \vec{b})$.

As a reference example, we consider the Siverts distribution. A Siverts contribution in agreement with the Trento definition[42] may be introduced in the unpolarized quark distribution of a transversely polarized proton as

$$q(x, k_x, 0) = q_U(x, k^2) + \frac{k_x}{M} q_S(x, k^2), \quad (18)$$

where we have assumed that the proton is fully polarized along the \hat{y} axis.

We may Fourier-transform this equation with respect to \vec{k} , for $k_y = 0$, and write the result in the form

$$q(x, \vec{k}) = \int d^2b e^{ib_x k_x} \left(Q_U(x, b^2) + ib_x \frac{\partial}{\partial(b^2)} Q_S(x, b^2) \right) \equiv \int d^2b e^{ib_x k_x} \left(Q_U(x, b^2) + ib_x \epsilon''(b^2) Q_S''(x, b^2) \right) \quad (19)$$

where $Q_{U,S}$ is the Fourier transform of $q_{U,S}$ with respect to $\exp(i\vec{b} \cdot \vec{k})$, and all Q-functions are real and \vec{b} -even.

If we apply the above eq.17 to the impact parameter dependent distribution, and explicitly separate the b_x -even and b_x -odd parts of ϵ :

$$\epsilon \equiv \epsilon'(b^2) + b_x \epsilon''(b^2), \quad (20)$$

$$Q_\epsilon(x, \vec{b}) \approx Q_0(x + i\epsilon, b^2) \approx Q_0(x, \vec{b}) + i\epsilon \frac{dQ_0(x, b^2)}{dx} \quad (21)$$

$$\equiv Q_0(x, b^2) + i\epsilon'(b^2) \frac{dQ_0(x, b^2)}{dx} + ib_x \epsilon''(b^2) \frac{dQ_0(x, b^2)}{dx}, \quad (22)$$

it is evident that a part of the frequency spectrum of the interaction operator contributes to the Sivers term in eq.18.

VI. CONCLUSIONS

In this work, we have considered the action of a set of factorization breaking interactions, whose frequency spectrum is regular and peaked near the condition of conserved x . We required the width of this peak to be small: $\delta x \approx \epsilon \ll 1$.

We have shown that for such interactions it is possible to retain the factorization scheme, and write the result of the calculation of a distribution function as a series expansion with respect to ϵ .

The zeroth-order term is the T-even and universal distribution function $q(x)$ corresponding to the complete absence of initial state interactions. T-even corrections to $q(x)$ appear at second order in ϵ .

The first order correction is a T-odd distribution function, detectable in angular even-odd interference terms. It cannot be stated to be universal, at this level of analysis.

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